A note on vacuum energy from the de Sitter spectrum

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Abstract. It is shown that a well-known relation between entropy of a system and its energy spectrum being applied to the early universe determines the present vacuum energy and the time scale on which this energy can manifest itself. Given the present vacuum energy, the relation imposes a constraint on the initial inflationary state.

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There is strong evidence in favor of an accelerating universe which might be due to a tiny cosmological constant [1]. This means that the universe will asymptotically tend to the de Sitter state. In accordance with the inflationary cosmology, such a state existed also in the distant past, although with much more vacuum energy. In this connection, an important question arises: is there a relation between these states? Even if it is the case, then why does the present vacuum energy density have the value it has, $\rho \sim 10^{-47}$ GeV^4? Where does such a small number come from? It is so small in comparison with the expected Planckian scale $\rho_P \sim 10^{76}$ GeV^4 that its relation with the early inflationary epoch is considered to be unlikely [2].

In this note, I want to point out to a possible relation between the energy spectrum of de Sitter space in the early universe and the present vacuum energy density.

My proposal rests on the conception of entropy. As is well known, the entropy of a system S, by definition, is the logarithm of the number of states with energy between E and $E + \delta E$. The width δE is some energy interval characteristic of the limitation in our ability to specify absolutely precisely the energy of a macroscopic system. It is equal in order of magnitude to the mean fluctuation of energy of the system. Dividing δE by the statistical weight $e^{S(E)}$, we obtain the mean separation between levels in the interval [3]

$$\Delta E = \delta E \, e^{-S(E)}. \tag{1}$$

This expression can be immediately applied to de Sitter space. As is well known, it is a thermodynamical system with a temperature and an entropy given by

$$T = \frac{H}{2\pi} \tag{2}$$

$$S = \frac{Horizon\ area}{4G} = \frac{\pi M_P^2}{H^2}\,, \tag{3}$$

where H is the Hubble constant. Taking into account the first law of thermodynamics of de Sitter space $dE = TdS$ [4, 5], expressed in terms (2) and (3), we can also define the specific heat

$$C_v = \left(\frac{\partial E}{\partial T}\right)_V = \frac{1}{2\pi\, G\, T^2} = \frac{2\pi M_P^2}{H^2}. \tag{4}$$

Then, since the the mean square fluctuation of energy is $\langle(\delta E)^2\rangle = C_v T^2$, it follows that

$$\delta E \sim M_P. \tag{5}$$

Thus

$$\Delta E \sim M_P\, e^{-S(E)}. \tag{6}$$

In accordance with Bohr's frequency condition, (6) should equal to some frequency. The only frequency in the problem is the rate of expansion, i.e. the Hubble constant H. Such a choice also agrees with the uncertainty principle $\Delta E\ \Delta t \sim 1$, where $\Delta t \sim H^{-1}$. Thus we obtain

$$M_P\, e^{-S(E)} \sim H. \tag{7}$$

Notice that the value of H on the right-hand side of (7) is, of course, not the same as that in the argument of the entropy $S(E)$ on the left-hand side. The value of H on the left-hand side is given as initial one. The value of H on the right-hand side, in contrast, must be found; it correspondents to a space whose rate of expansion is the same as the mean energy spacing (7). In order to distinguish these values, we shall write them with different indexes: H_i for a given (initial) value and H_f for a required (final) one.

Let us estimate H_f. Consider the early universe. Reasonable scale of inflation ranges from the Planck scale $M_P \sim 10^{19}$ GeV ($H_i \sim 10^{19}$ GeV) to the GUT scale $M_{GUT} \sim 10^{16}$ GeV ($H_i \sim 10^{13}$ GeV). In accordance with (3), it correspondents to the range of $S_i \sim 10^0 - 10^{12}$ ($H_i \sim 10^{19} - 10^{13}$ GeV). Now if we take $S_i \sim 10^{2.15}$ ($H_i \sim 10^{17.8}$ GeV) from the range and use (7), we obtain $H_f \sim 10^{-42}$ GeV corresponding $\rho_f \sim 10^{-47}$ GeV^4 ($H = \sqrt{\frac{8\pi G\rho}{3}}$). On the other hand, the finiteness of the de Sitter entropy indicates that spectrum of energy is discrete. The discreteness of the spectrum means that there is a typical energy spacing (7). It defines a new time scale $t_f \sim H_f^{-1}$ of order $\sim 10^{17}$ s. This is the time scale on which the discreteness of the spectrum can only manifest itself. Therefore, it is natural to identify H_f and ρ_f with the present Hubble constant and the vacuum energy density.

We can also identify the time t_f with the Poincaré recurrence time. The quantum Poincaré Recurrence theorem states [6]: given a system in which all energy eigenvalues are discrete, a state will return arbitrarily close to its initial value in a finite amount of time. These Poincaré recurrences generally occur on a time scale exponentially large

in the thermal entropy of the system. Thus we define the Poincaré recurrence time $t_r \equiv t_f = M_P^{-1} e^{S(E)}$. We can say that the universe returns to its initial point to within the mean energy spacing $\Delta E \sim 10^{-42}$ GeV in the Poincaré recurrence time $t_r \sim 10^{17}$ s; this process has became noticeable with the detection of the cosmic acceleration.

We can also apply (7) to the present universe. Using the current value of the entropy $S_i \sim 10^{120}$ we obtain $H_f \sim 10^{-119}$ GeV. But this gives us nothing interesting.

In this note we apply a well-known thermodynamical relation between entropy of a system and its energy spectrum (1) to the early universe. It turns out that, despite the absence of a theory of quantum gravity, one can relate the de Sitter state of the early universe with that of today's universe and determine the present vacuum energy and the time scale on which this energy can manifest itself. Moreover, as is seen from (7), such a transition from the inflationary vacuum energy to the present value is related to a quantum transition between the energy levels of the initial state. On the other hand, the relation imposes a constraint on the initial inflationary state. To get a universe with the vacuum energy density $\rho \sim 10^{-47}$ GeV^4 today, we should have the universe at the beginning with the following parameters.

- Hubble constant $H_i \sim 10^{17.8}$ GeV
- Entropy $S_i \sim 10^{2.15}$
- Energy $E_i \sim 10^{20.2}$ GeV
- Width of the spectrum $\delta E \sim 10^{19}$ GeV
- Mean energy spacing $\Delta E \sim 10^{-42}$ GeV.

That is, instead of asking why does the present vacuum energy density have the value $\rho \sim 10^{-47}$ GeV^4, we can ask why did the universe have the value $\rho_i \sim 10^{73.6}$ GeV^4 at the beginning.

References

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